Nonequilibrium Kondo transport through a quantum dot in a magnetic field

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We analyze universal transport properties of a strongly interacting quantum dot in the Kondo regime when the quantum dot is placed in an external magnetic field. The quantum dot is described by the asymmetric Anderson model with the spin degeneracy removed by the magnetic field resulting in the Zeeman splitting. Using an analytical expression for the tunneling density of states found from a Keldysh effective field theory, we obtain in the whole energy range the universal differential conductance and analytically demonstrate its Fermi-liquid and logarithmic behavior at low- and high-energies, respectively, as a function of the magnetic field. We also show results on the zero temperature differential conductance as a function of the bias voltage at different magnetic fields as well as results on finite temperature effects out of equilibrium and at a finite magnetic field. The modern nonequilibrium experimental issues of the critical magnetic field, at which the zero bias maximum of the differential conductance starts to split into two maxima, as well as the distance between these maxima as a function of the magnetic field are also addressed.

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I. INTRODUCTION

Among various quantum many-particle phenomena in condensed matter physics the Kondo effect[1] stands out as a unique fundamental state exhibiting remarkable properties both in bulk systems[2,3] and in a quantum dot (QD) setup[4–7].

In QDs[8] the Kondo effect appears when a QD is switched into the Coulomb blockade regime by tuning the gate voltage. Normally the QD states with even and odd numbers of electrons (Coulomb valleys) result in a zero bias minimum of the differential conductance as a function of the bias voltage. However, when the temperature T is lowered this situation changes qualitatively: the QD states with even numbers of electrons still result in a zero bias minimum of the differential conductance while for the QD states with odd numbers one observes an anomalous behavior where the differential conductance at zero bias shows a maximum often referred to as the Kondo resonance. This so-called zero bias anomaly is a manifestation of the unpaired spin inherent to the QD states with odd numbers of electrons.

Another important property of the Kondo effect is that together with its appearance the zero bias anomaly acquires universality: the differential conductance of different QDs collapses into a single function of the temperature and voltage. It depends on specific QD parameters only through the energy scale kT_{K}, where T_{K} is the Kondo temperature.

The Kondo effect has different universal character at low and high energies. At low energies, smaller than kT_{K}, one observes the Fermi-liquid behavior (strong coupling limit), a quadratic universal dependence of the differential conductance on the temperature, voltage, or external magnetic field. In the opposite regime of high energies, larger than kT_{K}, the differential conductance is characterized by a logarithmic universal dependence (weak coupling limit). At intermediate energies it shows a universal crossover between these two limits.

The zero bias anomaly and its universality have been observed in equilibrium[9] and nonequilibrium[10]. The nonequilibrium Kondo universality in the deep crossover is characterized by a universal value of the differential conductance at the Kondo voltage, $eV = kT_{\mathrm{K}}$. Theory predicts[11,12] that this universal value is equal to 2/3 of the maximum value which the differential conductance takes in equilibrium at zero temperature. This provides an efficient way to nonequilibrium experimental measurements of the Kondo temperature[13].

Interplay between the Kondo state and other quantum collective phenomena such as ferromagnetism[14], superconductivity[15,16] or the Kondo state in an external magnetic field[17,18] currently represent a very active research field both in theory and experiment.

The Kondo effect in a QD placed in an external magnetic field is of particular interest. The magnetic field couples to the spins of the electrons in the QD and produces the Zeeman splitting of the single-particle QD energy level. It is known[5–7] that such a magnetic field destroys the zero bias anomaly, that is, it restores the normal behavior where the QD states with odd numbers of electrons at low voltages result in minima of the differential conductance, similar to the QD states with even occupancies.

However, it turns out that the magnetic field does not destroy the universality. Theory[1] and experiment[18] demonstrate that even when there appears a minimum in the differential conductance at $V = 0$, it still remains universal both in equilibrium (as a function of T at $V = 0$) and nonequilibrium (as a function of T and V).

Important nonequilibrium issues which have been addressed in experiments[17,18] on the Kondo effect in an external magnetic field are 1) Kondo universality; 2) the critical magnetic field at which the zero bias maximum of the differential conductance starts to split into two maxima; 3) the distance between these maxima as a function

of the magnetic field; 4) the high-field limit of this distance. The issues above have obviously a highly nonequilibrium nature. The nonequilibrium theory of the Kondo effect in the *whole* energy range is a complicated problem even without a magnetic field. The numerical renormalization group (NRG) method[19] which is known to be a numerically exact tool in equilibrium cannot be easily generalized to nonequilibrium. To cover the whole energy range in nonequilibrium one has to resort to other methods. These are the non-crossing approximation (NCA) (intermediate and large energies)[5,20,21], equations of motion (mainly qualitative tool)[5,22,23], mean-field theories (low energies)[24,25], or $1/N$ expansions (low energies)[26]. In the whole energy range the real-time renormalization group method[11] is a powerful tool. It has been developed for the $s-d$ model[1]. To our knowledge, however, a generalization to include the Zeeman splitting has not been done yet. Another theoretical tool is the Keldysh effective field theory[12,27,28] developed for the Anderson model[29]. In its first application it has been shown that this theory well captures the weak coupling limit[27] and is capable to cope with finite, though large interaction strengths[28]. Recently it has been further developed to describe the behavior of the differential conductance in the whole range of temperatures and bias voltages[12].

In the present work we address the behavior of the Kondo state in an external magnetic field both in equilibrium and nonequilibrium. To this end we extend the Keldysh effective field theory[12] to account for the Zeeman splitting of the single-particle QD energy level and derive an approximate analytical expression for the QD tunneling density of states (TDOS). The theory is then applied to calculate the universal differential conductance as a function of the temperature, bias voltage and magnetic field in the whole energy range. We demonstrate that despite the approximations made in the theoretical derivation the theory has a number of advantages: 1) Fermi-liquid behavior when the Zeeman energy is much less than kT_{K} (advantage over NCA[1]); 2) logarithmic behavior when the Zeeman energy is much larger than kT_{K} (advantage over mean-field theories[1]); 3) critical magnetic field in a good agreement with previous theories[30] and experiments[17]; 4) distance between the maxima of the differential conductance as a function of the magnetic field in a good agreement with experiments[17]; 5) high-field limit of this distance in a good agreement with experiments[17]; 6) universality with the correct scaling given by T_{K}.

The paper is organized as follows. In Section II we present our model of a strongly correlated QD in a magnetic field. Here we also show the slave-bosonic transformation used in our work. The analytical expression for the QD TDOS resulting from an effective Keldysh field theory is given in Section III. Here we analytically investigate the low-energy sector of the theory proving its Fermi-liquid nature and providing the Fermi-liquid coefficients for the temperature, voltage and magnetic field behaviors as well as their universal ratios. Additionally, when the Zeeman energy is much larger than kT_{K} we analytically derive the logarithmic asymptotics of the differential conductance as a function of the magnetic field at $T=0$ and $V=0$. In Section IV, integrating the analytical expression for the QD TDOS, we calculate the universal behavior of the differential conductance in the whole range of temperatures, voltages and magnetic fields. Here we show all the regimes of the Kondo state: the strong coupling limit, weak coupling limit and the crossover region. Finally, we conclude the paper in Section V where all the advantages and drawbacks of our theory are summarized and a systematic way to improve the theory is mentioned.

II. THEORETICAL MODEL

To describe the strongly interacting QD we employ the single-impurity Anderson model[29] (SIAM). This model describes a system of two interacting electrons. The strength of the electron-electron interaction is given by the parameter $U>0$. In the absence of the interaction the electrons can occupy a spin degenerate single-particle energy level ϵ_{d}. We include the effect of an external magnetic field through the Zeeman splitting of this energy level, $\epsilon_{\mathrm{d}} \rightarrow \epsilon_\sigma$ with $\epsilon_\sigma \equiv \epsilon_{\mathrm{d}} + \sigma\Delta\epsilon/2$, $\sigma=\pm1$, $\Delta\epsilon \equiv g\mu_{\mathrm{B}}H$, where g is the g-factor, μ_{B} is the Bohr magneton and H is the magnetic field. The QD Hamiltonian is thus of the following form:

$$\hat{H}_{\mathrm{QD}} = \sum_\sigma \epsilon_\sigma \hat{n}_\sigma + U\hat{n}_\uparrow\hat{n}_\downarrow, \tag{1}$$

where the number operator $\hat{n}_\sigma$ is given in terms of the original Anderson fermionic operators as

$$\hat{n}_\sigma = d_\sigma^\dagger d_\sigma, \quad \{d_\sigma, d_{\sigma'}^\dagger\} = \delta_{\sigma,\sigma'} \quad \{d_\sigma, d_{\sigma'}\} = 0. \tag{2}$$

In addition to the interacting electronic system described by the SIAM, we also consider a noninteracting electronic system which models the contacts, that is an external system used to probe the QD. We consider two contacts and the corresponding Hamiltonian is

$$\begin{aligned} &\hat{H}_{\mathrm{C}} = \sum_{k,\sigma,x} \epsilon_{k\sigma} c_{k\sigma x}^\dagger c_{k\sigma x}, \\ &\{c_{k\sigma x}, c_{k'\sigma' x'}^\dagger\} = \delta_{k\sigma x, k'\sigma' x'}, \quad \{c_{k\sigma x}, c_{k'\sigma' x'}\} = 0, \end{aligned} \tag{3}$$

where k is the set of orbital quantum numbers in the contacts, σ is the contact spin degree of freedom and $x=\mathrm{L},\mathrm{R}$ denotes the left and right contacts. Here we also assume that the electronic states in the left and right contacts are characterized by the same complete set of quantum numbers, $\{k,\sigma\}$. The left and right contacts are in equilibrium states specified by the chemical potentials μ_{L} and μ_{R}, respectively.

The electrons can tunnel between the QD and contacts. These tunneling events are described in terms of

the tunneling Hamiltonian,

$$\hat{H}_{\mathrm{T}}=\sum_{k,\sigma,x}(T_{k\sigma}c^{\dagger}_{k\sigma x}d_{\sigma}+T^{*}_{k\sigma}d^{\dagger}_{\sigma}c_{k\sigma x}), \tag{4}$$

where $T_{k\sigma}$ are the tunneling matrix elements and we assume that these matrix elements are diagonal in the spin space and that they do not depend on x, that is, they are the same for the left and right contacts (QD symmetrically coupled to the contacts).

In equilibrium the chemical potentials of the QD and contacts are equal to each other. We denote this equilibrium value as μ_0 and define the applied bias voltage V as $\mu_{\mathrm{L,R}}=\mu_0\mp eV/2$.

As mentioned in the introduction, the Kondo effect, or the zero bias anomaly, develops at low temperatures out of the Coulomb blockade regime when the QD is blockaded with an odd number of electrons. Within the SIAM this means that the Kondo resonance takes place when the QD has one electron. This is achieved when the electron-electron interaction U exceeds the energy Γ (see below) resulting from the QD-contacts coupling.

To describe the relevant physics and at the same time to simplify the formalism, we consider the case of of the strongly interacting QD where $U\rightarrow\infty$. In this case the double occupancy is forbidden and the QD may have zero or one electron. At the same time the difference $\mu_0-\epsilon_{\mathrm{d}}$ is assumed to be finite but much larger than Γ so that the QD has one electron. This model admits the so-called slave-bosonic representation[1,31,32]. It represents a transformation where the original QD fermionic operators d_σ, $d^{\dagger}_{\sigma}$ are mapped onto new fermionic operators f_σ, $f^{\dagger}_{\sigma}$ and bosonic (often called slave-bosonic) operators b, $b^{\dagger}$ by means of the following relations:

$$\begin{aligned}&d_\sigma=f_\sigma b^{\dagger},\quad d^{\dagger}_{\sigma}=f^{\dagger}_{\sigma}b,\\&\{f_\sigma,f^{\dagger}_{\sigma'}\}=\delta_{\sigma,\sigma'},\,\{f_\sigma,f_{\sigma'}\}=0,\\&[b,b^{\dagger}]=1,\,[b,b]=0.\end{aligned} \tag{5}$$

Additionally, the operators f_σ and $f^{\dagger}_{\sigma}$ commute with the operators b, $b^{\dagger}$.

Physically the transformation (5) means that instead of using the states of the electrons in the QD one uses the states of the QD itself: the empty state (b, $b^{\dagger}$) and the state with one electron (f_σ, $f^{\dagger}_{\sigma}$).

Since the empty state and the state with one electron represent all the states of the QD, one arrives to the constraint,

$$f^{\dagger}_{\sigma}f_\sigma+b^{\dagger}b=\hat{1}, \tag{6}$$

which physically means that the total number of the new fermions and slave-bosons is restricted to be equal to one.

After the slave-bosonic transformation the QD and tunneling Hamiltonians become:

$$\hat{H}_{\mathrm{QD}}=\sum_{\sigma}\epsilon_\sigma f^{\dagger}_{\sigma}f_\sigma, \tag{7}$$

$$\hat{H}_{\mathrm{T}}=\sum_{k,\sigma,x}(T_{k\sigma}c^{\dagger}_{k\sigma x}f_\sigma b^{\dagger}+T^{*}_{k\sigma}f^{\dagger}_{\sigma}bc_{k\sigma x}). \tag{8}$$

The total Hamiltonian of this, in general nonequilibrium, problem is the sum of $\hat{H}_{\mathrm{QD}}$, $\hat{H}_{\mathrm{C}}$ and $\hat{H}_{\mathrm{T}}$ given by Eqs. (7), (3) and (8), respectively. This Hamiltonian together with the constraint in Eq. (6) represent the theoretical model able to describe the essential behavior of the zero bias anomaly arising due to the Kondo effect in the presence of an external magnetic field.

III. KELDYSH FIELD INTEGRAL SOLUTION AND ITS ASYMPTOTICS

As it is well known, the Kondo effect is an essentially nonperturbative phenomenon[1]. Therefore, to cover all the regimes of the Kondo state one must resort to nonperturbative methods able to deal with both electron-electron interactions and nonequilibrium.

In particular, the real-time formalism developed by Keldysh[33] represents a general and powerful method to treat interacting many-particle systems both in equilibrium and nonequilibrium. Originally this method was developed in the diagrammatic framework. Its field integral form has been developed in Ref. 34 to study the interplay between disorder and electron-electron interactions in metals. The advantage of the field integral form is that it reduces the problem to an analysis of a functional, the so-called Keldysh effective action. This analysis has a more systematic nonperturbative character than the diagrammatic selection on the basis of the topological structure reflecting the physical content of a given diagram.

In addition to being systematic, this nonperturbative approach turns out to be very general and applicable also to mesoscopic and QD systems. It has been applied, *e.g.*, to describe the Coulomb blockade in QDs[35,36] as well as the Kondo effect in the strong coupling limit[26], weak coupling limit[27,28] and in the whole energy range[12].

In particular, the slave-bosonic formulation presented in the previous section has been used in the Keldysh effective action framework in Refs. 12,27,28. The advantage of these slave-bosonic theories over slave-boson mean-field theories[1,25] and $1/N$ expansions[26] is that they take into account the constraint in Eq. (6) exactly while slave-boson mean-field theories and $1/N$ expansions account for this constraint only approximately. Because of this approximation one can only access the low energy (smaller than kT_{K}) physics of the Kondo state, that is the Fermi-liquid regime, or strong coupling limit, characterized by the quadratic dependence of the QD differential conductance. The main drawback of this approximation is that it does not lead to logarithmic terms[1] in the differential conductance and thus one cannot access intermediate (of order of kT_{K}) and high energy (larger than kT_{K}) physics, that is, the crossover region and weak coupling limit of the Kondo state, respectively. In contrast, the

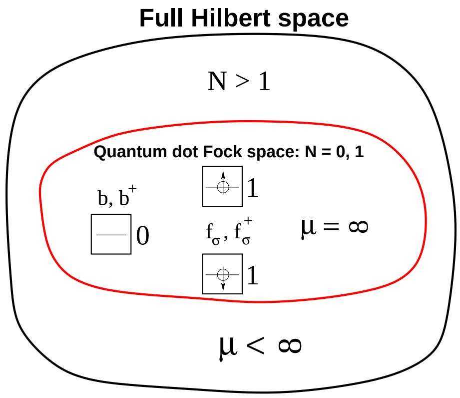

FIG. 1: (Color online) To formulate the field integral in Eq. (10) the constraint in Eq. (6) is temporarily removed by introducing a positive real parameter μ. When μ is finite, $0<\mu<\infty$, the QD may have any number of the new fermions and slave-bosons. One projects onto the physical subspace, the QD Fock space, by taking the limit $\mu\rightarrow\infty$ after the field integration has been performed.

slave-bosonic Keldysh effective action theories in Refs. 12,27,28 account for the constraint in Eq. (6) exactly and thus contain logarithmic terms.

As shown below, the formalism of Ref. 12 is straightforwardly generalized to the case of a QD in an external magnetic field.

The theory of Ref. 12 introduces a complex function E_α which is found from the condition that the QD TDOS,

$$\nu_\sigma(\epsilon)\equiv-\frac{1}{\pi\hbar}\mathrm{Im}[G^{+}_{\sigma\sigma}(\epsilon)], \tag{9}$$

where $G^{+}_{\sigma\sigma'}(\epsilon)$ is the QD retarded Green's function, in equilibrium and at zero temperature has a peak at $\epsilon=\mu_0$ and the amplitude of this peak is equal to the unitary limit.

In the case of a QD in an external magnetic field, one obtains the Keldysh effective action S_{eff} using a derivation similar to the one in Ref. 12 and expresses any physical observable $\hat{O}=\mathcal{F}(d^\dagger_\sigma,d_\sigma)$ as

$$\begin{aligned}&\langle\hat{O}\rangle(t)=\frac{1}{\mathcal{N}_0}\lim_{\mu\rightarrow\infty}e^{\beta\mu}\int\mathcal{D}[\bar{\chi},\chi]e^{\frac{i}{\hbar}S_{\mathrm{eff}}[\bar{\chi}^{\mathrm{cl,q}}(\tilde{t});\chi^{\mathrm{cl,q}}(\tilde{t})]}\times\\&\times\mathcal{F}[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)],\\&S_{\mathrm{eff}}[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)]=S_0[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)]+\\&+S_{\mathrm{T}}[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)],\end{aligned} \tag{10}$$

where $\chi^{\mathrm{cl,q}}$ are the classical and quantum[36] eigenstates of the bosonic annihilation operator b, $\beta\equiv1/kT$ is the inverse temperature and $\mathcal{N}_0$ is a normalization constant[27]. The limit $\mu\rightarrow\infty$ in Eq. (10) takes into account the constraint in Eq. (6). The point is that the field integral in Eq. (10) is most simply formulated for a system with arbitrary numbers of the new fermions and slave-bosons, that is temporarily removing the constraint in Eq. (6). One achieves such a formulation introducing a real positive parameter μ. As shown in Fig. 1, for $0<\mu<\infty$ the QD is allowed to have arbitrary numbers of the new fermions and slave-bosons, that is one deals with the full Hilbert space. One projects onto the physical QD Fock space by taking the limit $\mu\rightarrow\infty$.

In Eq. (10) $S_0[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)]$ is the standard free bosonic action[36] on the Keldysh contour and $S_{\mathrm{T}}[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)]=-i\hbar\,\mathrm{tr}\ln\bigl[-iG^{(0)-1}-i\mathcal{T}\bigr]$ is the tunneling action of the problem. Here the matrix $\mathcal{T}$ is off-diagonal in the QD-contacts space,

$$\mathcal{T}=\begin{pmatrix}0 & M^\dagger_{\mathrm{T}}(\sigma t|k'\sigma't')\\ M_{\mathrm{T}}(k\sigma t|\sigma't') & 0\end{pmatrix}, \tag{11}$$

$$\begin{aligned}&M_{\mathrm{T}}(k\sigma t|\sigma't')=\frac{\delta(t-t')\delta_{\sigma\sigma'}T_{k\sigma}}{\sqrt{2}\hbar}\times\\&\times\begin{pmatrix}\bar{\chi}^{\mathrm{cl}}(t)-\gamma_\sigma\sqrt{2} & \bar{\chi}^{\mathrm{q}}(t)\\ \bar{\chi}^{\mathrm{q}}(t) & \bar{\chi}^{\mathrm{cl}}(t)-\gamma_\sigma\sqrt{2}\end{pmatrix}.\end{aligned} \tag{12}$$

The Green's function matrix $G^{(0)}$ is block-diagonal in the QD-contacts space. Its QD block $G^{(0)}_{\mathrm{d}}$ has the standard 2×2 fermionic Keldysh structure:

$$G^{(0)}_{\mathrm{d}}(\sigma t|\sigma't')=\delta_{\sigma\sigma'}\begin{pmatrix}G^{+}_\sigma(t-t') & G^{\mathrm{K}}_\sigma(t-t')\\ 0 & G^{-}_\sigma(t-t')\end{pmatrix}. \tag{13}$$

In the frequency domain the components of the above matrix are

$$\begin{aligned}&G^{+}_\sigma(\omega)=\frac{\hbar}{\hbar\omega-(\epsilon_\sigma+\mu)+iE_{\alpha,\sigma}},\\&G^{-}_\sigma(\omega)=[G^{+}_\sigma(\omega)]^*,\\&G^{\mathrm{K}}_\sigma(\omega)=\frac{1}{2}[G^{+}_\sigma(\omega)-G^{-}_\sigma(\omega)]\sum_x\tanh\Bigl[\frac{\hbar\omega-\mu_x}{2kT}\Bigr].\end{aligned} \tag{14}$$

Here $E_{\alpha,\sigma}\equiv\alpha_\sigma\Gamma/2$, $\alpha_\sigma\equiv\gamma_\sigma\delta_\sigma$, $\Gamma\equiv2\pi\nu_{\mathrm{C}}|\tau|^2$ (ν_{C} is the contacts density of states, τ is the value of the tunneling matrix element $T_{k\sigma}$ assumed to be independent of k, σ).

In the case of the QD TDOS for spin σ the expression for the integrand in Eq. (10) is

$$\begin{aligned}&\mathcal{F}[\bar{\chi}^{\mathrm{cl,q}}(t);\chi^{\mathrm{cl,q}}(t)]=[\bar{\chi}_-(t)\chi_+(0)-\bar{\chi}_+(t)\chi_-(0)]\times\\&\times[G^{(0)-1}+\mathcal{T}]^{-1}(\sigma t|\sigma0),\end{aligned} \tag{15}$$

where $\bar{\chi}_\pm$, $\chi_\pm$ are the slave-bosonic fields on the forward and backward branches[36] of the Keldysh contour.

Anticipating that the Kondo physics arises from the spin-flip processes, to obtain the QD TDOS for fixed spin σ within the lowest order expansion (see below), we first choose in the σ- and $-\sigma$-blocks of the matrix $G^{(0)}_{\mathrm{d}}$ the imaginary parts of $E_{\alpha,\sigma}$ and $E_{\alpha,-\sigma}$ as $E^{\mathrm{I}}_{\alpha,\sigma}=E^{\mathrm{I}}_\alpha-\sigma\Delta\epsilon$ and $E^{\mathrm{I}}_{\alpha,-\sigma}=E^{\mathrm{I}}_\alpha$, while the real parts are $E^{\mathrm{R}}_{\alpha,\sigma}=E^{\mathrm{R}}_{\alpha,-\sigma}=E^{\mathrm{R}}_\alpha$. Here E_α is found, as in Ref. 12, at zero magnetic field, yielding $E^{\mathrm{R}}_\alpha/kT_{\mathrm{K}}\approx1.42$, $E^{\mathrm{I}}_\alpha/kT_{\mathrm{K}}\approx1.05$ with $kT_{\mathrm{K}}=2W\exp[-\pi(\mu_0-\epsilon_{\mathrm{d}})/\Gamma]$ (W is the Lorentzian width of the contacts density of states). After this choice the σ- and $-\sigma$-blocks of $G^{(0)}_{\mathrm{d}}$ are both

expressed through the spin $-\sigma$. Finally, as in Ref. 12, we expand $S_{\rm eff}$ and $\mathcal{F}$ up to the second order in the slave-bosonic fields in $\chi^{\rm cl}(t)-\delta_\sigma\sqrt{2}$, $\bar{\chi}^{\rm cl}(t)-\gamma_\sigma\sqrt{2}$ and $\chi^{\rm q}(t)$, $\bar{\chi}^{\rm q}(t)$.

The resulting QD TDOS obtained after performing the Gaussian field integral in Eq. (10) is then given by an expression similar to the one in Ref. 12 except for the spin dependence arising due to the Zeeman splitting,

$$\nu_\sigma(\epsilon)= \\ =\frac{1}{2\pi}\frac{\Gamma}{[\epsilon_{\rm d}+\sigma\Delta\epsilon/2-\epsilon+\Gamma\Sigma^{+}_{\sigma{\rm R}}(\epsilon)]^2+[\Gamma\Sigma^{+}_{\sigma{\rm I}}(\epsilon)]^2}, \tag{16}$$

$$\begin{aligned}&\Sigma^{+}_\sigma(\epsilon)=\\ &=\sum_x\Biggl\{\frac{1}{4\pi}\psi\left[\frac{1}{2}-\frac{W}{2\pi kT}\right]+\frac{1}{4\pi}\psi\left[\frac{1}{2}+\frac{W}{2\pi kT}\right]-\\ &-\frac{1}{2\pi}\psi\left[\frac{1}{2}+\frac{E_\alpha}{2\pi kT}-\frac{i\mu_x}{2\pi kT}+\frac{i(\epsilon-\sigma\Delta\epsilon)}{2\pi kT}\right]+\\ &+\frac{i}{2}\frac{1}{\exp\bigl(\frac{-\mu_x+iW}{kT}\bigr)+1}\Biggr\},\end{aligned} \tag{17}$$

where ψ is the digamma function.

Here we would like to mention the main drawback of the approximations made in deriving Eqs. (16) and (17). Due to the second order expansion of Eq. (15), the theory does not take into account all inelastic cotunneling processes which are of the fourth order in the tunneling matrix elements. These processes are included only partly, through the effective action. Therefore, the present theory underestimates $\nu_\uparrow(\epsilon)$ at energies $\epsilon-\mu_0>g\mu_{\rm B}H$ ($H>0$) and $\nu_\downarrow(\epsilon)$ at energies $\epsilon-\mu_0<-g\mu_{\rm B}H$. Furthermore, it is known[37,38] that for the $s-d$ model at very small magnetic fields ($g\mu_{\rm B}|H|\ll kT_{\rm K}$) the QD TDOS peak is located at $\epsilon=(2/3)g\mu_{\rm B}H$ and at $\epsilon=g\mu_{\rm B}H$ at larger fields. Our theory for the highly asymmetric SIAM predicts, as one can see from Eqs. (16) and (17), that the peak is always located close to $\epsilon=g\mu_{\rm B}H$. This perhaps can be attributed partly to the quality of our approximation and partly to the physical difference between the $s-d$ model and the highly asymmetric SIAM. It is technically complicated to include higher order terms in Eq. (15), and the corresponding theory will be the focus of our future research. However, below we demonstrate that already this simple theory has a number of advantages and is in a good agreement with some other theoretical predictions as well as with experiments.

With the QD TDOS (16) we can calculate the differential conductance using the expression[5,20] for the current through the QD,

$$\begin{aligned}&I=\frac{e}{\hbar}\sum_\sigma\int_{-\infty}^{\infty}d\epsilon[n_{\rm R}(\epsilon)-n_{\rm L}(\epsilon)]\frac{\Gamma}{4}\frac{W^2}{\epsilon^2+W^2}\nu_\sigma(\epsilon),\\ &n_{\rm L,R}(\epsilon)=\frac{1}{\exp[\beta(\epsilon-\mu_0\pm eV/2)]+1}.\end{aligned} \tag{18}$$

The differential conductance σ_d is obtained from Eq. (18) via the derivative of the current through the QD

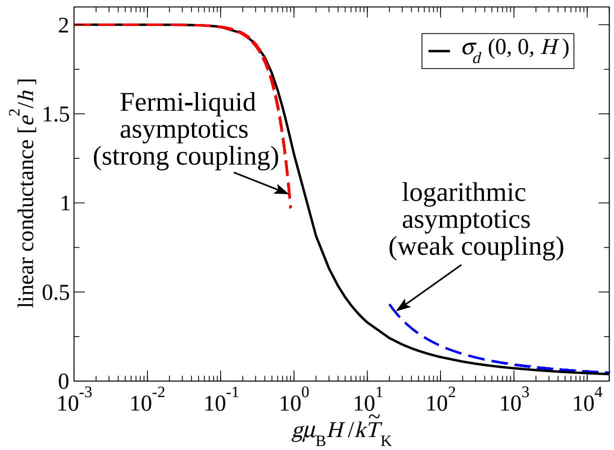

FIG. 2: (Color online) The QD universal differential conductance as a function of the magnetic field in the Kondo regime at $T=0$ and $V=0$. We use $\mu_0-\epsilon_{\rm d}=7\,\Gamma$, $W=100\,\Gamma$ which gives $kT_{\rm K}/\Gamma\approx5.63\cdot10^{-8}$. The Kondo temperature $\widetilde{T}_{\rm K}$ is defined in the text. The black solid curve shows the differential conductance obtained from Eqs. (16), (17) and (18). The dashed lines show the Fermi-liquid (red/upper) and logarithmic (blue/lower) asymptotics given by Eqs. (19) and (22), respectively.

with respect to the bias voltage, $\sigma_d=\partial I/\partial V$. The differential conductance is in general a function of the temperature, voltage and magnetic field, $\sigma_d=\sigma_d(T,V,H)$. It can be obtained substituting the QD TDOS, Eq. (16), into the expression for the current through the QD, Eq. (18). This is a difficult task because the QD TDOS (16) is already a complicated function which, in addition, must be integrated. Therefore, in general one has to perform a numerical integration to obtain the differential conductance.

However, at low energies it turns out to be possible to obtain the differential conductance analytically. From Eqs. (16), (17) and (18) it follows that at $T<T_{\rm K}$, $e|V|<kT_{\rm K}$ and $g\mu_{\rm B}|H|<kT_{\rm K}$ the differential conductance takes the form:

$$\begin{aligned}\sigma_d(T,V,H)=\frac{2e^2}{h}\Biggl[1-c_T\left(\frac{T}{T_{\rm K}}\right)^2-c_V\left(\frac{eV}{kT_{\rm K}}\right)^2-\\ -c_H\left(\frac{g\mu_{\rm B}H}{kT_{\rm K}}\right)^2\Biggr],\end{aligned} \tag{19}$$

which shows that at low energies the QD TDOS (16) leads to two important consequences. First, the differential conductance is a universal function of the temperature, bias voltage and magnetic field with the correct scaling given by the Kondo temperature $T_{\rm K}$ of the corresponding problem without magnetic field. Second, the differential conductance does not contain terms linear in T, V and H, which means that the QD TDOS (16) correctly reflects the low energy physics leading to the Fermi-liquid behavior.

The Fermi-liquid coefficients are given as

$$c_T = \frac{4}{3}\frac{2\ln(2|\mathcal{E}_\alpha|)+1}{|\mathcal{E}_\alpha|^2}, \quad c_V = \frac{1}{\pi^2}\frac{4\ln(2|\mathcal{E}_\alpha|)+1}{|\mathcal{E}_\alpha|^2},$$
$$c_H = \frac{4}{\pi^2}\frac{\ln(2|\mathcal{E}_\alpha|)+1}{|\mathcal{E}_\alpha|^2}, \tag{20}$$

with the following universal ratios

$$\frac{c_V}{c_T} = \frac{3}{2\pi^2}\frac{4\ln(2|\mathcal{E}_\alpha|)+1}{4\ln(2|\mathcal{E}_\alpha|)+2} \approx 0.13043,$$
$$\frac{c_H}{c_T} = \frac{3}{2\pi^2}\frac{2\ln(2|\mathcal{E}_\alpha|)+2}{2\ln(2|\mathcal{E}_\alpha|)+1} \approx 0.19509, \tag{21}$$

where $\mathcal{E}_\alpha \equiv E_\alpha/kT_{\mathrm{K}}$. For comparison we also give the universal ratios for the symmetric Anderson model, where one first puts $\mu_0 - \epsilon_d = U/2$ and only after that takes the limit $U \to \infty$: $c_V/c_T = 3/(2\pi^2) \approx 0.15198$, $c_H/c_T = 1/\pi^2 \approx 0.10132$.

In equilibrium and at zero temperature one obtains from Eqs. (16), (17) and (18) the differential conductance at $g\mu_{\mathrm{B}}|H| \gg kT_{\mathrm{K}}$:

$$\sigma_d(0,0,H) = \frac{2e^2}{h}\frac{\pi^2}{4}\frac{1}{\ln^2\big(\frac{g\mu_{\mathrm{B}}|H|}{kT_{\mathrm{K}}}\big)}. \tag{22}$$

As one can see, at large magnetic fields the differential conductance at $T = 0$ and $V = 0$ is also a universal function of the magnetic field with the correct scaling. Moreover, it correctly reflects the high energy physics leading to the logarithmic behavior[1].

Already these low- and high-energy asymptotics demonstrate advantages of the present theory over other slave-bosonic theories like mean-field theories or NCA which do not have the logarithmic or Fermi-liquid behavior, respectively[1].

In the next section we obtain the differential conductance from Eqs. (16), (17) and (18) in the whole energy range both in equilibrium and nonequilibrium. However, to obtain it in this case one has, in general, to perform the integration in Eq. (18) numerically.

IV. UNIVERSAL RESULTS IN THE WHOLE ENERGY RANGE

It has been demonstrated in Ref. 12 that the differential conductance resulting from the Keldysh effective action theory is a universal function of the temperature and bias voltage in the whole energy range. It has also been shown that it has the correct scaling given by the Kondo temperature T_{K}. Here we provide the differential conductance resulting from the Keldysh effective action theory taking into account an external magnetic field, Eqs. (16) and (17). Using Eq. (18), we obtain the linear conductance as a function of the magnetic field at $T = 0$, $\sigma_d(0,0,H)$. It is shown in Fig. 2. As one expects, in the Kondo regime $\sigma_d(0,0,H)$ turns out to be a universal function of $g\mu_{\mathrm{B}}H/kT_{\mathrm{K}}$ (or $g\mu_{\mathrm{B}}H/k\widetilde{T}_{\mathrm{K}}$, where $\widetilde{T}_{\mathrm{K}}$ is defined as $\sigma_d(\widetilde{T}_{\mathrm{K}},0,0) = \sigma_d(0,0,0)/2$, $\widetilde{T}_{\mathrm{K}} \approx 1.47\,T_{\mathrm{K}}$).

Let us now address the nonequilibrium Kondo physics in the presence of an external magnetic field. In Fig. 3 we show the zero temperature differential conductance as a function of the bias voltage at different magnetic fields. Again all the curves are universal with the scaling given by the Kondo temperature, which means that Eqs. (16) and (17) provide the differential conductance

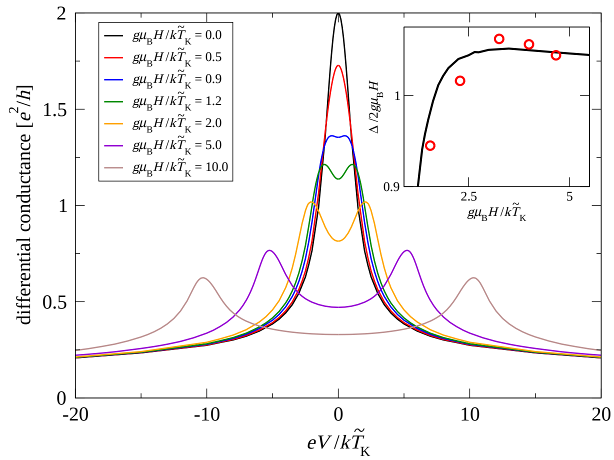

FIG. 3: (Color online) The universal differential conductance as a function of the bias voltage at zero temperature and different magnetic fields. The parameters are the same as for Fig. 2. When the magnetic field increases the zero bias anomaly first decreases and then starts to split into two peaks. This happens at the critical magnetic field $g\mu_{\mathrm{B}}H_c \approx 0.9\,k\widetilde{T}_{\mathrm{K}}$. The inset shows the distance between these two peaks as a function of the magnetic field. The circles in the inset show the experimental data from Ref. 17.

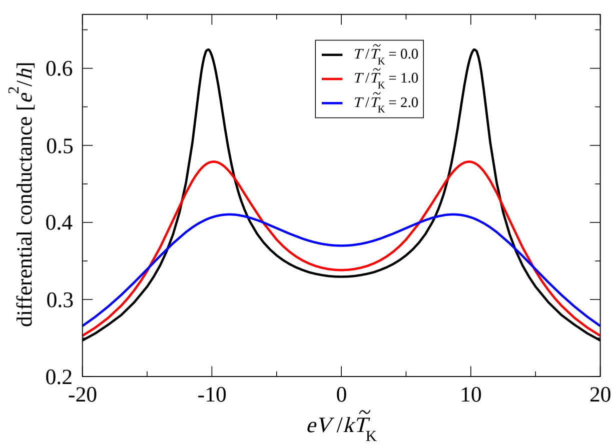

FIG. 4: (Color online) The universal differential conductance as a function of the bias voltage at finite temperatures and magnetic field $g\mu_{\mathrm{B}}H = 10\,kT_{\mathrm{K}}$. The parameters are the same as for Fig. 2. The increase in temperature leads to two effects: 1) the height of the peaks reduces; 2) the width of the peaks increases (broadening).

with the correct scaling also in nonequilibrium and finite magnetic fields. From Fig. 3 one observes that when the magnetic field is increased the zero bias Kondo peak first decreases and then splits into two peaks. Our theory predicts that the zero bias peak starts to split at the critical magnetic field $g\mu_{\mathrm{B}}H_c \approx 0.9\,k\widetilde{T}_{\mathrm{K}}$ which is in a good agreement with another theoretical estimate[30] predicting $g\mu_{\mathrm{B}}H_c \approx 1.0\,k\widetilde{T}_{\mathrm{K}}$. The inset of Fig. 3 shows the distance Δ between the two peaks as a function of the magnetic field. At low magnetic fields Δ has a rapid increase and then saturates at $\Delta/2g\mu_{\mathrm{B}}H \approx 1.017$. This high field limit of our theory is in a good agreement with the theoretical prediction made in Ref. 37. As one can see from the inset of Fig. 3, our theory is also in a good agreement with experiments in Ref. 17. In contrast, the theories in Refs. 30,37 both predict a saturation at much larger fields than observed in Ref. 17.

Finally, in Fig. 4 we show the differential conductance as a function of the bias voltage at finite temperatures and at the magnetic field value $g\mu_{\mathrm{B}}H = 10\,k\widetilde{T}_{\mathrm{K}}$. Also in this case the differential conductance is a universal function with correct scaling given by the Kondo temperature. As one expects, when the temperature increases the two peaks in the differential conductance are lowered and broadened.

V. CONCLUSION

In conclusion, we have applied the Keldysh effective action theory to study the Kondo effect in QDs in an external magnetic field both in equilibrium and nonequilibrium. To this end we have generalized the Keldysh effective action theory from Ref. 12 to take into account the effect of the magnetic field through the Zeeman splitting. An approximate analytical expression for the QD TDOS has been obtained. We have used this QD TDOS to calculate the differential conductance in the whole energy range and demonstrated that the theory has a number of advantages: 1) Fermi-liquid behavior when the Zeeman energy is much less than kT_{K} (advantage over the non-crossing approximation (NCA)[1]); 2) logarithmic behavior when the Zeeman energy is much large than kT_{K} (advantage over mean-field theories[1]); 3) critical magnetic field in a good agreement with previous theories[30] and experiments[17]; 4) distance between the maxima of the differential conductance as a function of the magnetic field in a good agreement with experiments[17]; 5) high-field limit of this distance in a good agreement with experiments[17]; 6) universality with the correct scaling given by T_{K}.

At the same time, due to the second order expansion of Eq. (15), the theory does not take into account all inelastic cotunneling processes which are of the fourth order in the tunneling matrix elements. Therefore, the present theory underestimates the differential conductance at voltages such that $e|V| > g\mu_{\mathrm{B}}|H|$ (see Fig. 3). An additional consequence of this approximation is that at high magnetic fields the differential conductance as a function of the bias voltage is oversensitive with respect to the temperature (see Fig. 4). These drawbacks can be eliminated by taking into account higher order terms in Eq. (15). This is technically more complicated and will be addressed in our future research.

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